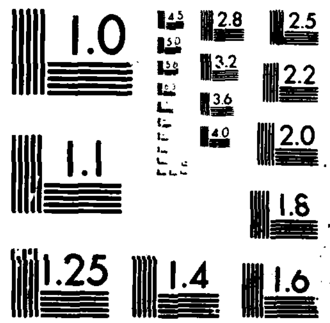
AD-A082 630 UNIVERSITY OF SOUTHERN CALIFORNIA LOS ANGELES DEPT O--ETC F/G 12/1
ESTIMATION AND IDENTIFICATION FOR MODELING DYNAMIC SYSTEMS.(U)
FEB 80 J M MENDEL AFOSR-75-2797
UNCLASSIFIED AFOSR-TR-80-0208 NL

1 OF 1
AD
A082630

END
DATE
FILMED
5-80
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AFOSR-TR- 80-0208

(12)

LEVEL III

February 20, 1980

A075059

ADA082630

DTIC ELECTE MAR 21 1980 D C

ESTIMATION AND IDENTIFICATION FOR MODELING DYNAMIC SYSTEMS

FINAL REPORT

AFOSR GRANT 75-2797

January 1, 1975 - December 31, 1979

1473

by

Jerry M. Mendel, Principal Investigator
Research Professor of Electrical Engineering
Department of Electrical Engineering
University of Southern California
Los Angeles, California 90007

80 3 20 087

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 18 19 AFOSR-TR- 80-0208	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 ESTIMATION AND IDENTIFICATION FOR MODELING DYNAMIC SYSTEMS AD75059		5. TYPE OF REPORT & PERIOD COVERED 9 Final Rept. 1 Jan 75-31 Dec 79
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Jerry M. Mendel		8. CONTRACT OR GRANT NUMBER(s) 15 AFOSR-75-2797
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Southern California, Dept. of Elect. Engineering-Systems, Los Angeles, CA 90007		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 16 17 61102F 2304 A2
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM, Bolling AFB, Washington, D.C. 20332		12. REPORT DATE 11 Feb [redacted] 80
		13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 361 560 12 25		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

algorithms, estiamting, identification, mathematical models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This document is the Final Report on AFOSR Grant 75-2797. It summarizes the research activity during the period January 1, 1975 - December 31, 1979 on the subjects of: multistage algorithms, identification from input/output data, modeling, estimation, and identification of layered media systems, and some miscellaneous topics.

TABLE OF CONTENTS

I. INTRODUCTION

Systems are modelled in order to understand and explain them better and as a prelude to action. Aircraft dynamics, for example, may be identified so that better designs can be made, or so that adaptive control actions can be taken. Our attention in this study has been directed at aspects of estimation and identification that are connected with system understanding.

This study has been aimed at three problem areas: multistage modeling, estimation, and identification algorithms for dynamical systems; parametric identification algorithms for estimation of input-output response; and, state estimation and parameter identification for a new class of models -- <u>causal functional equations</u> -- which describe wave propagation in layered media systems.

The first problem is concerned with developing estimation algorithms both for parameter and state estimation that are recursive in the dimension of the parameter vector or state vector. Such algorithms will find utility in system modeling work, where model dimension is often a variable.

The second problem is concerned with identifying a linear system's input-output response rather than a parametric representation for the sytem. The basic idea is that parameter estimation techniques often give very poor estimates of a system's response; hence, we are interested in new algorithms that lead to good estimates of system input-output response.

The third problem is concerned with developing whole new theories of state estimation and parameter identification for causal functional equations. These equations are continuous-time, linear, time-invariant and contain multiple time delays. They do not contain derivatives or integrals, and no literature apparently exists for them. Causal functional equations are applicable to diverse areas such as reflection seismology, transmission lines, speech processing, optical thin coatings and EM problems.

This study, performed from January 1, 1975 through December 31, 1979, led to 32 publications.

II. SUMMARY OF RESEARCH EFFORT

We summarize our research activities for the three problem areas mentioned in Section I in Paragraphs A, B, and C below. Some tangential results are also summarized in Paragraph D.

A. Multistage Algorithms

1. Multistage least-squares parameter estimation algorithms have been extended to multistage weighted least-squares algorithms [1]. This extension permits one to do multistage unbiased minimum variance estimation of parameters in a linear model, which broadens the multistage philosophy to more than one estimation technique.

2. Multistage least-squares algorithms have been applied to a number of interesting problems associated with identification of a sampled impulse response [1].* Suppose a test signal, $u(t)$, is applied at $t=t_0$ to a linear, time-invariant, causal, but unknown system whose output, $y(t)$, and input are measured. The unknown impulse response is to be identified using sampled values of $u(t)$ and $y(t)$. For such a system

$$y(t_k) = \int_{t_0}^{t_k} w(\tau)u(t_k - \tau)d\tau \; ; \tag{1}$$

or, if we assume that (a) $w(t) \approx 0$ for all $t \geq t_w$, (b) $[t_0, t_w]$ is divided into N equal intervals, each of width ΔT, and (c) for $\tau \in [t_{i-1}, t_i]$, $w(\tau) \simeq w(t_{i-1})$, and $u(t - \tau) \simeq u(t - t_{i-1})$, then

*Publications under this grant are listed in Section III. References [e.g.[A]] are listed at the end of the report.

$$y(t_k) \simeq \sum_{i=1}^{N} w_1(t_{i-1})u(t_k - t_{i-1}) \tag{2}$$

where

$$w_1(t_{i-1}) = \Delta T \; w(t_{i-1}) \; . \tag{3}$$

It is straightforward to identify the N unknown parameters $w_1(t_0)$, $w_1(t_1),\ldots,\; w_1(t_{N-1})$ via least-squares; however, for N to be known t_w must be accurately known and ΔT must be chosen judiciously.

In actual practice t_w is not known that accurately so that N may have to be varied. Multistage least-squares estimators (LSE's) have been used to handle this situation in a computationally efficient manner.

Sometimes, ΔT can be too coarse for certain regions of time, in which case significant features of w(t), such as a ripple, may be obscured. In this situation, we would like to "zoom-in" on those intervals of time and re-discretize y(t) just over those intervals, thereby adding more terms to $y(t_k)$. We have shown how to use multistage LSE's to zoom-in on regions of time in a very effective manner.

3. Computational requirements for multistage LSE's have been studied [2]. Our approach was to count the total number of multiplications associated with our different multistage algorithms. Total number of multiplications is a good indicator of total computation time. We have compared the computation time requirements for our multistage algorithms with one-stage (conventional) LSE algorithms from two different points of view.

In order to illustrate the different points of view, recall that by means of n-stage LSE's it is possible to simultaneously obtain least-squares estimates for parameters in n models. According to the first point of view we treat the n-stage LSE as a recursive procedure for obtaining just the least-squares estimate of n-vector $\underline{\theta}$; hence, the computation time required to obtain $\hat{\underline{\theta}}$ by means of the n-stage LSE should be compared with the computation time required to obtain $\hat{\underline{\theta}}$ by means of a one-stage LSE. By this first point of view we are comparing computation time requirements on an absolute basis. According to the second point of view we treat the

n-stage LSE as a recursive procedure for obtaining least-squares estimates for n different linear models; hence, the computation time required to obtain $\hat{\underline{\theta}}$ by means of the n-stage LSE should be compared with the computation time required to obtain least-squares estimates for each one of the n linear models, each estimate being obtained by a one-stage LSE. By the second point of view we are comparing computation time requirements on the basis of equal information.

The following conclusions have been reached (quantitative conclusions are given in [2]): For batch data processing, it is always more efficient to use multistage LSE's. This is true both on an absolute and equal information basis. For sequential data processing, conclusions depend on whether we choose to use a "standard" LSE or a "stabilized" LSE. Often, for purposes of accuracy, one must use a stabilized LSE, in which case it is again always more efficient to use multistage LSE's. If, on the other hand, accuracy is not so important, then it is usually more efficient to use one-stage standard LSE's than multistage standard LSE's.

4. Friedland's [A] bias filtering technique, which is an example of a multistage Kalman/Bucy (K/B) filter for a very special plant (or transition) matrix and process covariance matrix, has been extended to the following class of nonlinear systems [3]:

$$\dot{\underline{x}}(t) = \underline{f}[\underline{x}(t), t] + \Lambda_1\underline{b} + \underline{w}(t) \tag{4a}$$

$$y(t) = \underline{h}[\underline{x}(t), t] + \Lambda_2\underline{b} + \underline{v}(t). \tag{4b}$$

In this system, states $\underline{x}$ enter nonlinearly; but, biases enter linearly. Signals $\underline{w}(t)$ and $\underline{v}(t)$ are zero mean white noise processes.

5. In our efforts to generalize Friedland's [A] decomposition to the situation of a bias vector described by a first-order Markov process, we have had to rederive his decomposition by a route different from the one taken by him. His approach was to augment the bias states to the dynamical $\underline{x}$ states and to show that the Riccati equation for the augmented system

could be decomposed so that $\hat{\underline{x}}(k)$ can be computed via the following equation:

$$\hat{\underline{x}}(k) = \tilde{\underline{x}}(k) + V_x(k)\hat{\underline{b}}(k) \tag{5}$$

where $\tilde{\underline{x}}(k)$ is the bias-free estimate of $\underline{x}(k)$, computed as if no biases were present, $\hat{\underline{b}}(k)$ is the optimum estimate of the bias, and $V_x(k)$ is a matrix, that is computed recursively, which blends the estimates of $\hat{\underline{x}}(k)$ and $\hat{\underline{b}}(k)$ together to give $\hat{\underline{x}}(k)$, the bias corrected estimate of $\underline{x}(k)$. Our approach [4,5], which is constructive in nature, is to assume the existence of the decomposition in Eq. (5), for which we require that $\hat{\underline{x}}(k)$ and $\hat{\underline{b}}(k)$ be unbiased estimators of $\underline{x}(k)$ and $\underline{b}(k)$, and, that the trace of the estimation error covariance matrix for $\hat{\underline{x}}(k)$ be minimum. We then demonstrate that the resulting $\hat{\underline{x}}(k)$ is indeed the minimum variance estimator of $\underline{x}(k)$; hence, our assumed decomposition is valid.

Besides obtaining all of Friedland's results (and, giving some new physical meaning to some of the matrix quantities such as $V_x(k)$), we also show that $\hat{\underline{b}}(k)$ in Eq. (5) is a minimum variance estimator of $\underline{b}$ for an auxiliary parameter estimation problem; hence, we have demonstrated that it is possible to add or remove elements from $\underline{b}$ so that $\underline{x}(k)$ and the modified bias vector can be reestimated without having to redo all the calculations. This is important in applications where the bias states are used to model constant instrumentation error sources [B]; for, it can happen that not all of the error sources are significant, so that some of them should be deleted from the final filter, or, that significant error sources may have been initially neglected, and should be included in the final filter.

6. Washburn [6,7] has generalized Friedland's [A] bias estimation technique to partitioned dynamical systems. In the general case, the calculations are of the dimension of the overall system, so that, except for some special but important cases, there are no computational advantages to the multistage approach. Those special cases, where there does appear to be computational advantages for the multistage approach are: colored noise and weak coupling between the partitioned systems.

The general results are important in themselves, since they provide the theory for a particular decomposition of the optimal state estimator for a system of possibly large dimension (i.e., a large scale system). This decomposition gives added insight into the structure and performance of the minimum variance unbiased estimator. In addition, the methodology of proof for this multistage decomposition provides a means for investigating other decompositions of interest.

B. Identification From Input/Output Data

1. Several existing identification algorithms have been analyzed to determine how well the resulting models match the input/output response of the true system, especially with respect to stability. It has been found that in the identification of autoregressive processes the standard least-squares algorithm, (which is also maximum likelihood) gives good results. This would be expected from the properties of maximum likelihood methods, and the inherent numerical robustness of the algorithm. This robustness is due to the fact that a Toeplitz matrix is inverted which is generally positive definite and well-conditioned. However, in the identification of autoregressive moving average processes a number of algorithms first estimate the Markov parameters (e.g., [C], [D]) and then invert the associated Hankel matrix. It has been found that these algorithms do not perform well in their reproduction of the input/output response for two reasons. Firstly, because only a small number of Markov parameters is used, there is high sensitivity to errors, and secondly, the Hankel matrix may become almost singular (Glover [E]). These results have been verified by simulation for the method given in [C].

2. Preliminary work on input design for identification [8] has indicated that the choice of design criterion can be critical. An illustrative example was produced where the trace of the information matrix was maximized subject to an input energy constraint as follows. There are two identifiable parameters to be identified and the available input energy must be shared between both. The above criterion however indicated that all the input energy should be channelled into identifying

the more easily identified parameter. This resulted in one parameter being identified very well and no information being obtained on the other parameter, which is clearly not desired.

3. Several deterministic identification problems have been solved [9]. The general problem is: given a finite set of finite length input/output sequences from an unknown discrete-time system, derive a minimal order linear dynamical model with appropriate initial conditions that would produce the observed input-output sequences. Previous approaches to this problem (e.g. [F]) have been based on involved matrix manipulations and have required certain restrictive assumptions on the data. The new method is based on the geometric theory of linear systems and is completely general, in that an arbitrary number of input/output sequences of arbitrary length are allowed, and the initial conditions can be either zero or free. The resulting algorithm is simply stated in terms of recursive subspace equations and includes tests for the identifiability of the data. It has further been shown that several other problems in linear systems (e.g. observers and exact model matching) can be formulated within the same framework.

4. Substantial progress has been made in implementing and evaluating approximate realization methods (i.e., the approximate realization of given impulse response sequence by a low order linear system). Given exact data there are a large number of different algorithms that will produce exact realizations, and given approximate data the same methods can be approximately implemented to produce approximate realizations. This is similar in spirit to an equation error method and a selection of six approximate realization methods of this type have been implemented and have been evaluated using experimental impulse response data.

C. Modeling, Estimation, and Identification of Layered Media Systems

1. We have developed time-domain state space models for lossless layered media which are described by the wave equation and boundary conditions [10-13]. Our models are for non-equal one-way travel times; hence, they are more general than existing models of layered media which

are usually for layers of equal one-way travel times. Full state models, which involve 2K states for a K-layer media systems, as well as half-state models, which involve only K states have been developed and related. Certain transfer functions, which appear in the geophysics literature in connection with models of layered media with equal travel times have been generalized to the situation of non-equal travel times. Our state space models represent a new class of equations, <u>causal functional equations</u>, for which we have not been able to find any literature. These equations are continuous-time, linear, and contain multiple time-delays. Their impulse response is an infinite sequence of non-uniformly spaced impulse functions.

2. We have <u>proven</u> the truth [11,14] of the following decomposition of the solutions to the lossless wave equation in layered media: the complete output from a K-layer media system, which is comprised of the superposition of primaries, secondaries, tertiaries, etc., can be obtained from a single state space model of order 2K -- the complete model -- or from an infinite number of models, each of order 2K, the output of the first of which is just the primaries, the output of the second of which is just the secondaries, etc. This decomposition of the solution to the lossless wave equation into physically meaningful constituents (i.e., primaries, secondaries, etc.) is called a canonical Bremmer Series decomposition, after Bremmer, who in 1951 established a similar decomposition [G].

In many geophysical situations, where reflection coefficients are quite small, the decomposition can be truncated after secondaries or tertiaries; hence, it also represents a way to approximate the solution to the wave equation.

We have made connections [15] between our state space models and the integral equations given by Bremmer [G] for generating the partial residuals.

We have shown how to go from Bremmer's integral equations to our state equations by assuming a medium with a wave number that has finite jumps (discontinuities) which occur at the interfaces and that is constant within a layer. These assumptions for wave number are associated with what we mean by a horizontally layered homogeneous earth.

We have also demonstrated that Bremmer's integral equations can be obtained by the W. K. B. method which gives approximate solutions to second-order differential equations [H]. This justifies earlier claims [11,14] that the Bremmer series decomposition can be used to approximate the complete solution to the lossless wave equation by truncating that decomposition after a small number of terms.

3. We have developed [16] a general theory for describing reinforced events between multiple reflections in lossless layered media, which are described by the wave equation and boundary conditions (e.g., horizontally stratified nonabsorptive earth with vertically traveling plane compressional waves).

Reinforcements occur whenever two or more multiple reflections from different paths inside the media arrive at the surface at the same time so that they add (positively or negatively) together. Those reinforcements occur regardless of what the travel time is in each layer, and distort the appearance of a seismogram; for, they lead one to believe that a significant event has occurred by the appearance of a large amplitude segment of the seismogram, whereas, in reality, that large event is a sum of (many) smaller events.

Our general theory is applicable to a K-layer media system with nonuniform travel times and gives information about the exact location in time, number, and amplitude of reinforced events for n-aries (i.e., secondaries, tertiaries, etc.), where n = 1, 2, 3 ... . The starting point for the development of this theory is Mendel's Bremmer series decomposition [11,14] and the operator description of state space models of layered media [12] by means of which n-ary reflections (where n = 1, 2, 3, ...) are generated and analyzed separately and related to each other. The two most significant multiple reflections, secondaries, and tertiaries, have been studied extensively. We have demonstrated that not only do reinforcements occur between the same kind of multiple reflections (e.g., between secondaries), but that reinforcements also occur <u>across</u> different kinds of multiple reflections (e.g., between secondaries and tertiaries).

4. Because our causal functional state space models for a layered media system represent a new class of equations, we have had to study the computer simulation of these equations. Two computational methods have been considered [17]. In the first approach, we discretized the time axis and inserted states of intermediate delays, to arrive at a set of standard finite-difference equations. For our particular system, matrix multiplications can be reduced to simple scalar multiplications. In the second approach, we defined mapping rules for the transformation of states at an interface, and kept a state reference table for look-up and branching. The procedure is similar to ray-tracing. Several experiments have been performed to show the trade-off between storage requirement and CPU time-spent for the two methods.

5. We have developed a procedure for extracting reflection coefficients from noisy data [18] which we feel is a substantial generalization of similar procedures which have been reported in the literature ([I] for example). Associated with these earlier procedures are Standard Assumptions and Steps which include requirements that the data be noise free and that the observed seismic data be deconvolved. Our procedure avoids these restrictive requirements. Furthermore, our procedure totally avoids the concepts of z-transforms, minimum phase, spectral factorization, etc., which appear in the literature on this subject.

6. An important special case of a causal functional equation (CFE), occurs when all one-way travel times are equal. In this case the uniform CFE (UCFE) is

$$\underline{x}(t + \tau) = A\,\underline{x}(t) + \underline{b}\,m(t) \tag{6}$$

with initial values

$$\underline{x}(\sigma) \qquad \sigma \in [0, \tau) \triangleq \mathcal{T} \tag{7}$$

Recognizing that any time t ($t \in R$) can be expressed as

$$t = t' + M\tau \text{ where } t' \in \mathcal{T} \text{ and } M \text{ is an integer} \tag{8}$$

we have shown [19] that the solution to UCFE (6) is

$$\underline{x}\,[t' + (k+1)\tau] = A^{k+1}\,\underline{x}(t') + \sum_{i=0}^{k} A^{k-i}\, m(t' + i\tau) \tag{9}$$

where $t' \in \mathcal{T}$, $k = 0, 1, 2, \ldots$, and $t = t' + (k+1)\tau$.

Equation (9) explicitly shows how the state at any time $t = t' + (k+1)\tau$ depends on an initial condition $\underline{x}(t')$ and the input m. It is of interest to note that $\underline{x}(t)$ depends only on a <u>single element</u> of the initial values $\underline{x}(\sigma)$ $(\sigma \in \mathcal{T})$, namely $\underline{x}(t')$, and a finite number of point values of m. This shows that <u>the solution to the uniform causal functional state equation, although continuous-time in nature, derives its values in a discrete-time fashion for a given fixed value of $t' \in \mathcal{T}$</u>. Of course, there are an uncountable number of points in $\mathcal{T}$; hence <u>we can imagine</u> $\underline{x}(t)$ <u>as being generated by an uncountable number of discrete-time systems</u>.

When we simulate our results on a digital computer, computations are made every T sec. at discrete time points. Consequently, on a digital computer, $\underline{x}(t)$ is generated by a finite number of discrete-time systems which operate in parallel.

7. We have derived [19] the minimum-variance state estimator for UCFE

$$\underline{x}(t + \tau) = A\,\underline{x}(t) + B\,\underline{m}(t) + \underline{w}(t) \tag{10}$$

and its associated measurement equation

$$\underline{y}(t) = H\,\underline{x}(t) + \underline{n}(t) \tag{11}$$

Let $\hat{\underline{x}}(t)$ denote the minimum-variance estimate of $\underline{x}(t)$ based on all measurements in $\{\underline{y}(\lambda): 0 \le \lambda \le t, t \in R\}$. We have shown that for any fixed $t' \in \mathcal{T} = [0, \tau)$, $\hat{\underline{x}}(t)$, where $t = t' + M\tau$ $(M = 1, 2, \ldots)$, is given by the usual discrete-time Kalman filter equations (Ref. J, for example) with t' considered the initial starting time. Of course, to obtain $\hat{\underline{x}}(t)$

for all $t \in R$ we would need an uncountable number of discrete-time Kalman filters; but, imposing a mesh on $\mathcal{T}$ (with grid size equal to data sampling rate) leads to a finite number of Kalman filters which operate in parallel. To the best knowledge of the author this is the first estimation theory result that has led to a natural form of parallel data processing.

8. We have developed an extended minimum-variance estimator for simultaneous estimation of states and parameters (i.e., reflection coefficients) in a UCFE [20]. Simulation results were very disappointing. Reasons for the disappointing results are explained in [20].

9. A simple inverse filter has been developed [21] to suppress multiple reflections from a normal incidence synthetic seismogram. The filter was developed by means of Mendel's Bremmer Series Decomposition [14] and the operator description of state space model of layered media [12], and is given in terms of z-transforms as

$$\tilde{Y}_1(z) = \frac{Y(z)}{1 - \dfrac{r_0}{1 - r_0^2}\,\dfrac{Y(z)}{M(z)}} \tag{12}$$

In this equation $Y(z)$ is the synthetic seismogram measured by a sensor located on the surface. That sensor receives reflected signals from a layered media due to the normal incident input $M(z)$ applied at the same surface. Parameter r_0 is the reflection coefficient of the surface. $\tilde{Y}_1(z)$ is the output of the filter; it consists of the primary reflection portion of the seismogram and some residual terms; i.e., $\tilde{Y}_1(z) = Y_1(z) + \gamma(z)$. In general, the residual terms $\gamma(z)$ is relatively quite small, and $\tilde{Y}_1(z)$ is a good approximation of $Y_1(z)$. This filter is especially effective when r_0 is relatively large (as in most geophysical situations) in which case $\gamma(z)$ is almost negligible compared with $Y_1(z)$. The filter requires knowledge of the input waveform $M(z)$, surface reflection coefficient, r_0, and measured seismogram $Y(z)$. Observe that (12) represents a <u>nonlinear processing</u> of the seismogram $Y(z)$.

10. We have extended our normal incidence state space model to the non-normal incidence case [22]. The non-normal incidence (NNI) state space model is structurally the same as the normal incidence state space model except that it has twice as many state variables. Because of mode conversion in non-normal incidence, the scalar upgoing and downgoing waves and travel times in each layer as well as reflection and transmission coefficients in each interface are replaced by a vector of upgoing and downgoing waves, a vector of travel times, and matrices of reflection and transmission coefficients.

With this NNI model, we are able to generate synthetic seismograms for a plane wave source, and more importantly, for a two-dimensional point source.

11. We have developed a maximum-likelihood procedure for estimating both the reflection coefficients and one-way travel times [23] for a lossless layered media system in which the layers are non-equally spaced (in time). It uses a state space model as its starting point, one that is more general than (6) since now τ is different for each layer (i.e., $\tau_1 \neq \tau_2 \neq \tau_3 \neq \dots \tau_k \neq \tau$). The only source of uncertainty is measurement noise, n(t). Maximum-likelihood estimates of the parameters are obtained in a layer-recursive format. In essence, first r_1 and τ_1 are determined and layer 1 is stripped away; then r_2 and τ_2 are determined and layer 2 is stripped away; etc. To the best of our knowledge, this is the first time that both reflection coefficients and travel times have been simultaneously estimated in an optimal manner.

12. We have surveyed approaches to solving inverse problems for lossless layered media systems [24].

13. A Kunetz equation is often used as the starting point in the development of solutions for the inversion of one-dimensional, noise-free, normal incident seismograms, for which $|r_0| = 1$ (r_0 is the surface reflection coefficient). We have demonstrated a need for a Kunetz-type equation in which filtered signals can be used, so that noise effects (which are always present in real data) can be reduced. Furthermore, we

have shown that an infinite number of Kunetz-type equations exist for the lossless wave equation in layered media. Finally, we have shown that it is indeed valid to formulate and solve the inverse problem using filtered signals [25].

14. We have developed a unified procedure for estimating reflection coefficients of lossless layered media systems from noise seismic data [26]. Our procedure is a generalization of work described in [18] to the cases of source and sensor either at the surface or in the first layer (e.g., a water layer). We have handled these cases simultaneously, since, conceptually at least, they are similar.

Our procedure is to: (1) write state equations which describe the temporal evolution of all upgoing and downgoing signals; (2) derive an autoregressive-like equation for basement signal $d_{K+1}(t)$; (3) establish a parameter estimation problem for estimating the coefficients in the $d_{K+1}(t)$ equation, and generate the Normal equations which result from solution of that problem; (4) express the Normal equations in terms of measurable signals, through use of a Kunetz equation; and (5) study conditions for which the Normal equation can be solved, as well as properties of the solution.

Our procedure is constructive and during the study of its five steps we have provided answers to the following questions: (1) Why do we direct our attention at an equation for the unmeasurable basement signal, $d_{K+1}(t)$, rather than the measurable seismogram signal?; (2) What is the real role of the Kunetz equation?; (3) Where in the procedure is it necessary to distinguish the cases of source and sensor at the surface versus source and sensor in the first layer?; (4) Why is deconvolution necessary?; (5) Why is the solution limited to cases for which the amplitude of the surface reflection coefficient is unity?; and (6) Why is the solution so sensitive to noise?

15. We have summarized our work on uniform causal functional equations (UCFE) [27], (10) and (11). Because a UCFE is isomorphic to an uncountable number of discrete-time systems, each one initialized at $t' \in \mathcal{T} = [0, \tau]$, we have been led to a very simple proof of a widely used fact that UCFE (10) is properly initialized by $\underline{x}(\sigma)$, $\sigma \in \mathcal{T}$.

16. We have continued work on Habibi-Ashrafi's [23] maximum-likelihood procedure for estimating reflection coefficients and detecting one-way travel times [28] (in all of our other studies travel time τ is assumed known, and all layers have the same travel time). We are especially interested in making connections between his results and our suboptimal results [26], when his results are specialized to the case of uniform travel time.

17. We have developed a Bremmer series decomposition for the NNI case and have also designed approximate filters for suppression of multiple reflections [29,30]. These filters are based on the algebraic structure of the NNI Bremmer series decomposition.

D. Miscellaneous

1. It is common in system modeling that a number of parameters are not known precisely but will be determined later from empirical data or by subsequent design decisions. It is therefore important to study such structured systems, and in [31] we have derived necessary and sufficient conditions for the structural controllability of multi-input structured linear systems. The methods used in [31] are substantially simpler than previous approaches and rely on the properties of the interconnection of sub-systems which is appropriate in the study of large scale systems.

2. We have demonstrated [32] that the spacing parameter, which appears in many stochastic approximation identification algorithms ([K] and [L], for example) is unnecessary. Those algorithms, written as

$$\underline{\hat{\theta}}(k+\ell) = \underline{\hat{\theta}}(k) + \ldots$$

where ℓ is the spacing parameter, and k is chosen as an integer multiple of ℓ, should be written as

$$\underline{\hat{\theta}}(k+1) = \underline{\hat{\theta}}(k) + \ldots$$

where $k = 0, 1, \ldots$.

III. PUBLICATIONS UNDER GRANT 75-2797

[1] J.M. Mendel, "Multistage Least-Squares Parameter Estimators," IEEE Transactions on Automatic Control, Vol. AC-20, pp. 775-782, December 1975.

[2] J.M. Mendel, "Computational Requirements for Multistage Least-Squares Estimators," presented at Sixth Symposium on Nonlinear Estimation and its Applications, San Diego, California, September 1975.

[3] J.M. Mendel, "Extension of Friedland's Bias Filtering Technique to a Class of Nonlinear Systems," IEEE Transactions on Automatic Control, Vol. AC-21, pp. 296-298, April 1976.

[4] J.M. Mendel and H.D. Washburn, "Multistage Estimation of Bias States," presented at the 1976 IEEE Conference on Decision and Control, Clearwater, Florida, December 1976.

[5] J.M. Mendel and H.D. Washburn, "Multistage Estimation of Bias States in Linear Systems," Int. J. on Control, Vol. 28, pp. 511-524, 1978.

[6] H.D. Washburn, "Multistage Estimation and State Space Layered Media Models," Ph.D. Dissertation, March 1977.

[7] H.D. Washburn and J.M. Mendel, "Multistage Estimation of Dynamical and Weakly Coupled States in Continuous-Time Linear Systems," IEEE Transactions on Automatic Control, April 1980.

[8] M.S. Grewal and K. Glover, "Relationships between identifiability and Input Selection," IEEE Conference on Decision and Control, Houston, Texas, December 1975, pp. 526-528.

[9] E. Emre, L.M. Silverman, and K. Glover, "Generalized Dynamic Covers for Linear Systems with Applications to Deterministic Identification and Realization Problems," presented at the IFAC Symposium on Large Scale Systems, Udine, Italy, 1976, and published in IEEE Transactions on Automatic Control, Vol. AC-22, pp. 26-36, February 1977.

[10] N.E. Nahi and J.M. Mendel, "A Time-Domain Approach to Seismogram Synthesis for Layered Media," presented at the 46th Annual Meeting of the Society of Exploration Geophysicists, Houston, Texas, October 1976.

[11] J.M. Mendel, "A Cannonical Bremmer Series Decomposition of Solutions to the Lossless Wave Equation in Layered Media," presented at the 1977 Joint Automatic Control Conference, San Francisco, California, June 1977.

[12] J.M. Mendel, N.E. Nahi, L.M. Silverman, and H.D. Washburn, "State Space Models of Lossless Layered Media," presented at the 1977 Joint Automatic Control Conference, San Francisco, California, June 1977.

[13] J.M. Mendel, N.E. Nahi and M. Chan, "Synthetic Seismogram Using the State Space Approach," Geophysics, Vol. 44, pp. 880-895, May 1979.

[14] J.M. Mendel, "Bremmer Series Decomposition of Solutions to the Lossless Wave Equation in Layered Media," IEEE Transactions on Geoscience Electronics, Special Issue on Seismic Data Processing, April 1978.

[15] J.M. Mendel and F. Aminzadeh, "On the Bremmer Series Decomposition of Solutions to the Lossless Wave Equation in Layered Media," Geophysical Prospecting, April 1980.

[16] J.M. Mendel and J.S. Lee, "Reinforcement of Reflections," presented at the 47th Annual International Meeting of the Society of Exploration Geophysicists, Calgary, Canada, September 1977.

[17] W.M. Chan, N.E. Nahi, and J.M. Mendel, "Computational Solutions to a Non-Uniform Time-Delay Linear System," presented at Symposium on Applications of Computer Methods in Engineering, University of Southern California, Los Angeles, California, August 1977.

[18] N.E. Nahi, J.M. Mendel and L.M. Silverman, "Recursive Derivation of Reflection Coefficients from Noisy Seismic Data," presented at 1978 IEEE Int'l. Conf. on Acoustics, Speech, and Signal Processing, Tulsa, Oklahoma, April 1978.

[19] H.D. Washburn and J.M. Mendel, "Minimum-Variance State Estimation for Uniform Causal Functional Equations," presented at 17th IEEE Conference on Decision and Control, San Diego, California, January 1979.

[20] J.M. Mendel, "Identification of Reflection Coefficients from Noisy Data by Means of Extended Minimum Variance Estimators: A Critical Examination," presented at 17th IEEE Conference on Decision and Control, San Diego, California, January 1979.

[21] J.S. Lee and J.M. Mendel, "Suppression of Multiple Reflections," presented at 48th Annual International Meeting of the Society of Exploration Geophysicists, San Francisco, November 1978.

[22] F. Aminzadeh and J.M. Mendel, "Non-Normal Incidence State Space Model," presented at 48th Annual International Meeting of the Society of Exploration Geophysicists, San Francisco, November 1978.

[23] F. Habibi-Ashrafi, "Estimation of Parameters in Lossless Layered Media Systems," Ph.D. Dissertation, University of Southern California, October 1978.

[24] Mendel, J.M., and Habibi-Ashrafi, F., 1979, "A Survey of Approaches to Solving Inverse Problems for Lossless Layered Media Systems," presented at the 5th IFAC Symposium on Identification and Parameter Estimation, Darmstadt, Federal Republic of Germany, Sept. 24-28, 1979.

[25] Mendel, J.M., 1979, "Generalized Kunetz-Type Equations," Geophysical Prospecting, 1980.

[26] Mendel, J.M., 1979, "Suboptimal Estimation of Reflection Coefficients, for Lossless Layered Systems, from Noisy Data," presented at 49th Annual Meeting of the Society of Exploration Geophysicists, New Orleans, La.

[27] Mendel, J.M., and Washburn, H.D., 1979, "On Uniform Causal Functional Equations," submitted for publication.

[28] Habibi-Ashrafi, F., and Mendel, J.M., 1979, "Simultaneous Estimation of Reflection Coefficients and Travel-Times in Lossless Layered Media Systems," submitted for publication.

[29] Aminzadeh, F., "Non-Normal Incidence State Space Model for Layered Media Systems," Ph.D. Dissertation, Sept., 1979.

[30] Aminzadeh, F. and Mendel, J.M., 1979, "Suppression of Surface Multiples in a Non-Normal Incidence Plane Wave Seismogram," presented at 49th Annual Meeting of the Society of Exploration Geophysicists, New Orleans, La.

[31] K. Glover, L.M. Silverman, "Characterization of Structural Controllability," IEEE Transactions on Automatic Control, Vol. AC-21, No. 4, August 1976.

[32] I. Gustavsson, "The Spacing Problem in Stochastic Approximation Algorithms," unpublished notes.

IV. PROFESSIONAL PERSONNEL ASSOCIATED WITH RESEARCH EFFORT

1. Prof. Jerry M. Mendel, Principal Investigator, 1975-1979.
2. Prof. Keith Glover, Senior Investigator, 1975 and part of 1976; Visiting Research Scholar, Summer 1977.
3. Prof. Leonard Silverman, 1976.
4. Dr. A. Ivar B. Gustavsson, Lund Institute of Technology, Division of Automatic Control, Lund, Sweden; Visiting Research Scholar, Summer 1976 (one month).
5. Dr. H.D. Washburn, Ph.D. student who received his degree in June 1977. Thesis title is "Multistage Estimation and State Space Layered Media Models."
6. Dr. F. Habibi-Ashrafi, Ph.D. student who received his degree in Jan. 1979. Thesis title is "Estimation of Parameters in Lossless Layered Media Systems."
7. Dr. F. Aminzadeh, Ph.D. student who received his degree in June 1979. Thesis title is "Non-Normal Incidence State Space Models for Layered Media Systems."
8. D. Kuan, J.S. Lee, M. Shiva, M. Chan, and S. Kundu, Ph.D. students.
9. J. Biddle, K. Saedinia and A. Yu, computer programmers.

V. INTERACTIONS

The following papers which are listed in Section III were presented at major technical conferences: [2], [4], [8], [9], [10], [11], [12], [16], [17], [18], [19], [20], [21], [22], [24], [26], [30].

Additionally, Dr. Mendel presented the following talks, which were based (all, or in part) on research supported by this grant:

1. "Multistage Least-Squares Parameter Estimation: An Approach to Modeling Large Scale Systems," presented to Systems Engineering/ Operations Research Seminar at the Univ. of California, Irvine, May 29, 1974.

2. "State Space Models of Layered Media," presented to Systems Science Seminar, Univ. of California, San Diego, Feb. 2, 1977.

3. "Estimation of Reflection Coefficients for Lossless Layered Systems," Univ. of Houston, E.E. Dep't. Seminar, March 5, 1979; EE Systems Seminar, USC, April 16, 1979; and Eindhoven Univ. of Technology, Eindhoven, The Netherlands, Oct. 2, 1979.

VI. NEW DISCOVERIES AND SPECIFIC APPLICATIONS STEMMING FROM THE RESEARCH EFFORT

No patents were obtained. The following represent new discoveries: (1) generalization of Friedland's [A] bias estimation technique to partitioned dynamical systems [6, 7]; (2) state space models for layered media systems [10, 11, 12, 13, 22]; (3) a state space Bremmer series decomposition [11, 14, 29]; and (4) a maximum-likelihood procedure for estimating both the reflection coefficients and travel times for a lossless layered media system [23, 28].

All of our work described in Section II.C is applicable to reflection seismology for oil and gas exploration.

REFERENCES

A. B. Friedland, "Treatment of Bias in Recursive Filtering," IEEE Trans. on Automatic Control, Vol. AC-14, pp. 359-367, August 1969.

B. R. Barton and A. Ljungwe, "Trajectory Reconstruction Utilizing Kalman Filter Estimation Theory," AIAA Paper 71-933, presented at 1971 AIAA Guidance, Control, and Flight Mechanics Conference, Hempstead, New York, August 1971.

C. R.K. Mehra, "The Identification of Linear Dynamic Systems With Applications to Kalman Filtering," IEEE Trans. on Automatic Control, Vol. AC-16, No. 1, February 1971.

D. G.N. Saridis and R.N. Lobbia, "Parameter Identification and Control of Linear Discrete-Time Systems," IEEE Trans. on Automatic Control, Vol. AC-17, No. 1, pp. 52-60, 1972.

E. K. Glover, "Some Geometrical Properties of Linear Systems With Implications in Identification," VI Triennial World Congress of IFAC, Boston, August 1975.

F. B. Gopinath, "On the Identification of Linear Time-Invariant Systems from Input-Output Data," BSTJ, Vol. 48, pp. 1101-1113, 1969.

G. H. Bremmer, "The WKB Approximation as the First Term of a Geometric-Optical Series," The Theory of Electromagnetic Waves, A Symposium, Interscience Publishers, New York, pp. 169-179, 1951.

H. R. Bellman and R. Kalaba, "Functional Equations, Wave Propagation, and Invariant Imbedding," J. Math. and Mech., Vol. 8, pp. 683-704, 1959.

I. E.A. Robinson and S. Treitel, "The Spectral Function of a Layered System and the Determination of the Waveforms at Depth," Geophysical Prospecting, 1977.

J. J.S. Meditch, "Stochastic Optimal Linear Estimation and Control," McGraw-Hill, New York, 1969.

K. G. Saridis and R.N. Lobbia, "Parameter Identification and Control of Linear Discrete-Time Systems," IEEE Trans. on Automatic Control, Vol. AC-17, pp. 52-60, 1972.

L. H.J. Kushner, "A Simple Iterative Procedure for the Identification of the Unknown Parameters of a Linear Time-Varying Discrete System," Preprints 1962 Joint Automatic Control Conference.